

# Cold Tests and Modeling of Low- $Q$ Quasi-Optical Resonators

Richard P. Fischer, *Member, IEEE*, and Arne W. Fliflet, *Member, IEEE*

**Abstract**—Measurements are reported on low- $Q$  open resonators at 85 GHz with a large centered coupling hole in each mirror. This is the first study of open resonators with large diffraction losses and efficient input coupling at millimeter wavelengths. The input resonator for a quasi-optical gyrokylystron is described with quality factor  $Q = 2000$  and 13% round-trip losses where approximately 65% of the incident power in the waveguide is coupled to the fundamental  $\text{TEM}_{00}$  mode. Two models have been developed to calculate the coupling from the rectangular waveguide through a circular aperture into the open resonator. Both models properly account for the increased diffraction loss of the resonator when coupling apertures are present. Measured results are in reasonably good agreement with values calculated using both scalar diffraction theory and an equivalent-circuit model.

**Index Terms**—Cold tests, coupling holes, diffraction theory, Fabry–Perot resonators, millimeter-wave resonators,  $Q$  factor.

## I. INTRODUCTION

THE quasi-optical gyrotron (QOG) is under development as a high-power source of millimeter-wave radiation. The output resonator of this device is formed by a pair of spherical mirrors separated by many radiation wavelengths, where the round-trip diffraction losses are small (1%–3%). A recent quasi-optical gyrokylystron experiment successfully used an open resonator with a low quality factor ( $Q$ ) to prebunch the electron beam, which allowed the device to operate as an oscillator, amplifier, mode-locked oscillator, and phase-locked oscillator [1], [2]. The input resonator is designed with relatively small mirrors and large coupling holes so that the round-trip losses are large (13%) while providing efficient coupling from the waveguide to the fundamental Gaussian mode. The behavior of these resonators is much different than the more familiar low-loss high- $Q$  Fabry–Perot resonators.

Open resonators operating at millimeter wavelengths have been studied for many years for applications such as dielectric characterization, filters, diplexers, and power combiners [3], [4]. Power is typically coupled into the resonator through small holes, slots, meshes, or dielectric launchers. Most applications require a large  $Q$  factor where the diffraction and coupling losses are small [5]. An extensive series of cold tests is reported in [6] for high- $Q$  quasi-optical resonators with small coupling holes, where good agreement is obtained between measured and calculated  $Q$  factors. The quasi-optical gyrokylystron

requires a low- $Q$  input resonator to prevent oscillations when the electron beam is present. An open resonator with small mirrors and large diffraction losses will provide the required  $Q$  factor. Large coupling holes are required to efficiently couple power from the drive source into the resonator. The theoretical effect of coupling apertures on the mode structure and losses of confocal open resonators has been presented [7], [8]. However, little experimental or theoretical work on low- $Q$  open resonators has been reported at millimeter wavelengths.

This paper first discusses the diffraction and aperture losses of low- $Q$  quasi-optical resonators using computer calculations based on scalar diffraction theory. Section III describes a model of coupling from a rectangular waveguide to the quasi-optical resonator which is based on the diffraction theory results. Here, it is shown that even small coupling holes cause increased diffraction at the mirror edge, which alters the fundamental behavior of coupling to a quasi-optical resonator from that of the more familiar closed cavity. Transmission measurements are performed on large circular apertures in rectangular waveguide at 85.5 GHz to help model the coupling to the resonator. A second model is then described, where an equivalent circuit is used to derive a closed-form expression for the external  $Q$  of the resonator based on the effective aperture reactance of the large coupling hole. In Section IV, some qualitative cold-test measurements are presented and compared to the coupling behavior of a closed cavity. A specific quasi-optical resonator is then considered where reflection, transmission, and mode pattern measurements are performed and the input coupling is optimized. Transmission and reflection measurements of the resonator are in relatively good agreement with values calculated using the two coupling models.

## II. LOW $Q$ OPEN RESONATORS

The  $Q$  of a Fabry–Perot-type resonator can be written  $Q = 4\pi L/\lambda f_L$ , where  $L$  is the separation between the mirrors,  $\lambda$  is the free-space wavelength, and  $f_L$  is the fractional roundtrip loss of the radiation in the resonator. In practice, this loss factor includes diffraction losses, coupling hole losses, and ohmic losses. For the low- $Q$  input resonator, ohmic losses are small and can be neglected. The diffraction/coupling  $Q$  is calculated with a computer code, which is based on the scalar formulation of Huygens's principle.<sup>1</sup> The integral equations of the open resonator are solved as a matrix eigenvalue problem, yielding the eigenfunctions. Outputs from the code include the diffraction/coupling losses and the electric-field distribution along the surface of the mirrors for the modes of interest.

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The authors are with the Beam Physics Branch, Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375 USA.

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<sup>1</sup>Code written by K. Yoshioka, with a formulation similar to that found in [9].

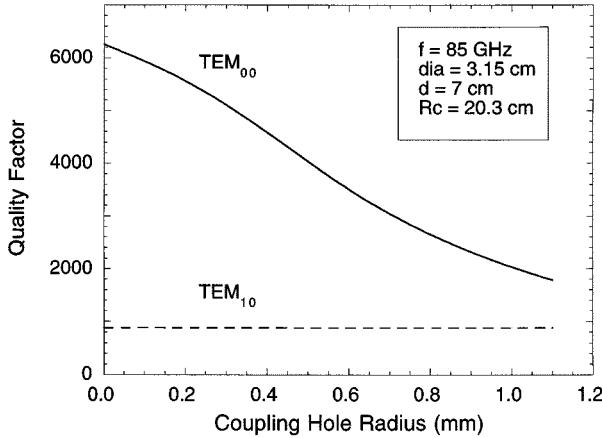


Fig. 1. Calculated variation of  $Q$  factor with coupling hole radius.

The calculated effect of a large coupling hole on a relatively low- $Q$  open resonator is shown in Fig. 1. The resonator is symmetric with 3.1-cm-diameter mirrors, 20-cm radius of curvature, and 7-cm separation. As the coupling hole radius is increased to 1 mm, the  $Q$  factor of the fundamental  $\text{TEM}_{00}$  mode decreases from 6600 to 2100. However, the pattern is still approximately Gaussian and the losses are less than those of competing modes. The next higher radial mode, the  $\text{TEM}_{10}$ , has a null on axis and is largely unaffected by the coupling hole. If the hole radius is increased beyond 1 mm, the coupling loss of the fundamental transverse mode approaches the diffraction loss and the mode becomes distorted. Under these conditions, a field minimum occurs at the center of the mirror and the mode pattern begins to resemble the  $\text{TEM}_{10}$  [7].

#### A. Diffraction Theory Model of Resonator Coupling

Consider a conventional closed cavity with input and output coupling holes and unloaded  $Q$  factor  $Q_0$ . The power reflection coefficient at port 1 can be written

$$R = \left( \frac{1 - \beta_1 + \beta_2}{1 + \beta_1 + \beta_2} \right)^2. \quad (1)$$

The coupling parameter  $\beta_i = Q_0/Q_{ei}$ , where  $Q_{ei}$  is the external (or coupling)  $Q$  of aperture  $i$ . The transmission coefficient from ports 1 to 2 is [11]

$$T = \frac{4\beta_1\beta_2}{(1 + \beta_1 + \beta_2)^2}. \quad (2)$$

To model the coupling to a quasi-optical resonator we assume that only the  $\text{TEM}_{00}$  mode is excited in the resonator. A more complete model would include the effects of higher order modes in the waveguide and resonator, as well as radiation scattered into free space. For small coupling holes, the transverse mode pattern is approximately Gaussian and is largely unaffected by the aperture. However, even small holes perturb the mode pattern so that diffraction losses at the edge of the mirror increase. This increased diffraction can be conveniently accounted for by introducing a third coupling parameter  $\beta_3 = (f_d - f_{d0})/f_{d0}$ . Here,  $f_d$  ( $f_{d0}$ ) is the diffraction loss of the res-

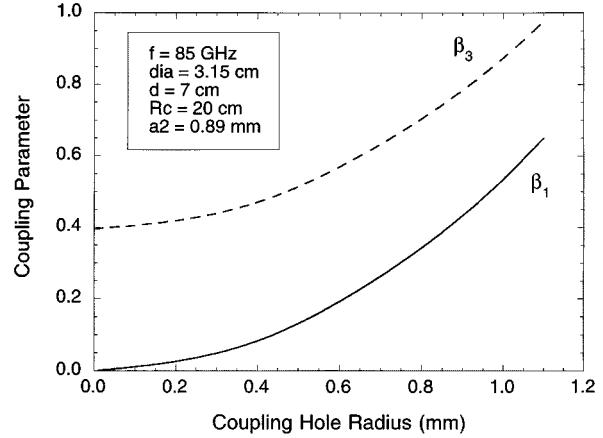


Fig. 2. Calculated coupling parameters as a function of coupling hole radius.

onator with (without) coupling holes. The reflection and transmission coefficients of the resonator may now be written

$$R = \left( \frac{1 - \beta_1 + \beta_2 + \beta_3}{1 + \beta_1 + \beta_2 + \beta_3} \right)^2 \quad (3)$$

$$T = \frac{4\beta_1\beta_2}{(1 + \beta_1 + \beta_2 + \beta_3)^2}. \quad (4)$$

The loaded  $Q$  of the resonator is given as  $Q_L = Q_0/(1 + \beta_1 + \beta_2 + \beta_3)$ . (Note that an alternative method of describing the open resonator is to allow the unloaded  $Q$  factor to vary with coupling hole size. This technique eliminates the need for  $\beta_3$  and yields equivalent results for reflection and transmission using (1) and (2). However, this notation is not physically intuitive since  $Q_0$  now varies with  $Q_{ei}$ .)

Consider a quasi-optical resonator similar to that in Fig. 1, except the output coupling hole is fixed at 1.8-mm diameter. Fig. 2 shows the calculated effect on  $\beta_1$  and  $\beta_3$  of increasing the input hole size while holding the output aperture constant. The input and output coupling hole radii are denoted as  $a_1$  and  $a_2$ , respectively. For  $a_1 = 0$  mm, there is no hole loss through aperture 1, but  $\beta_3 = 0.4$  due to the increased diffraction caused by the output hole. As the input hole diameter is increased, there is a increase in  $\beta_3$ , which is comparable to the loss through aperture 1. This effect has been noted by a number of researchers and demonstrates why the maximum coupling efficiency from a centered hole is approximately 50% [9], [10]. The increase in diffraction loss can be approximated as  $\beta_3 \simeq \beta_1 + \beta_2$ , provided that the input hole diameter is not too large.

It is instructive to compare the coupling behavior of a closed cavity to that of a quasi-optical resonator. The reflection, transmission, and cavity power are plotted in Fig. 3(a) as a function of  $\beta_1$  for a closed cavity. The output coupling parameter for this example is fixed at  $\beta_2 = 0.01$  ( $\beta_2 \ll \beta_1$ ). As expected, the power coupled to the cavity is maximum near critical coupling ( $\beta_1 = 1$ ), where the reflected power falls to zero. A much different result is obtained for the quasi-optical resonator in Fig. 3(b). The output coupling parameter is again held constant at  $\beta_2 = 0.01$ , and it is assumed that  $\beta_3 = \beta_1 + \beta_2$ . The reflected power is now monotonically decreasing with  $\beta_1$ , while the power coupled to the resonator mode continues to increase. This is quite different from the closed cavity result and makes

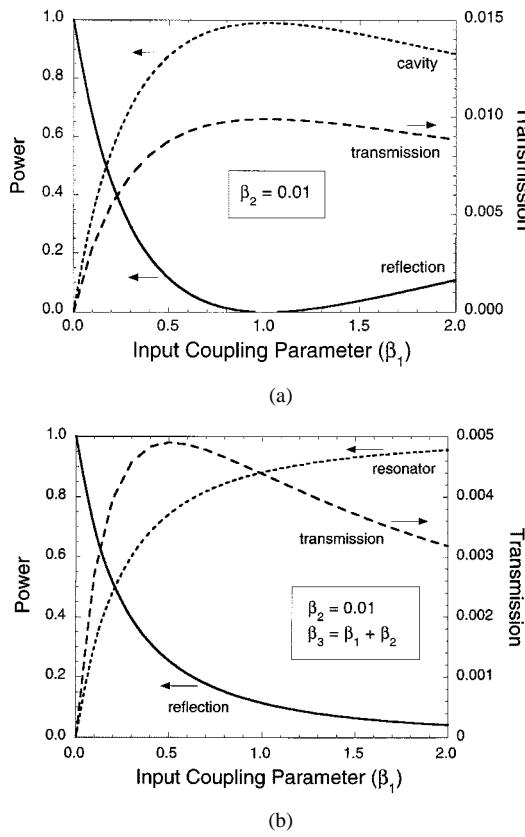


Fig. 3. Reflection, transmission, and coupled power for: (a) a closed cavity and (b) an open resonator.

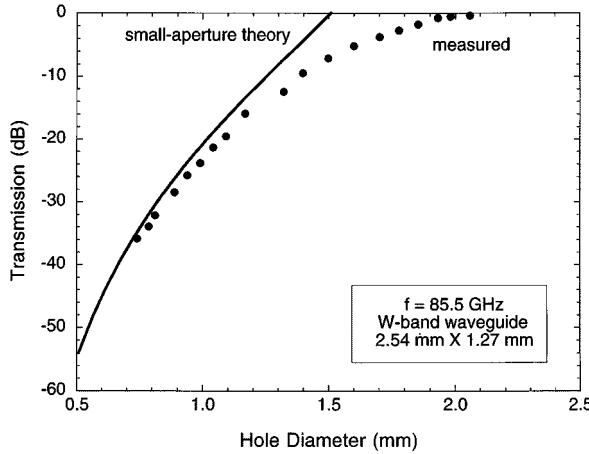


Fig. 4. Measured and calculated transmission through a circular aperture in the endwall of a rectangular W-band waveguide.

measurements of quasi-optical resonators more difficult to interpret. Under these conditions, the resonator does not show the critical coupling or overcoupling behavior that is common in closed cavities. If the input hole diameter continues to increase such that  $\beta_1 \gg 1$ , the diffraction loss will tend to increase faster than the coupling loss ( $\beta_3 > \beta_1 + \beta_2$ ). In this regime, radiation scattered from the resonator is increasingly important and the assumptions of this simple model are not valid.

The diffraction theory code, which calculates the resonator losses, assumes that all radiation incident upon the aperture is lost from the resonator [8]. A realistic model of coupling

through the aperture should include the effects of finite wall thickness and the cutoff property of the hole. For a frequency of 85.5 GHz, the  $TE_{1,1}$  circular waveguide mode will propagate for a hole diameter of 2.06 mm. Analytic formulas can be used to describe the transmission through a small aperture in a transverse wall in the waveguide. However, the approximations involved are not valid for large coupling holes (hole diameter  $\geq$  narrow wall of waveguide), thus, new measurements are required. Measured results are shown in Fig. 4 for the transmission of 85.5-GHz radiation through a circular aperture in a transverse wall of thickness 0.43 mm. The solid curve is the analytic small-aperture theory of [12], which includes the effect of the wall thickness and the large aperture. Note that the dimensions of standard W-band waveguide are  $2.54 \times 1.27 \text{ mm}^2$ .

One of the goals of this section is to modify the results of the diffraction theory code using the aperture transmission measurements to obtain new values for  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$ . Reducing the transmission through the input and output coupling holes will proportionately decrease the coupling parameters  $\beta_1$  and  $\beta_2$ . In this paper, we assume that reducing the aperture losses results in a corresponding decrease in  $\beta_3$ . This assumption is justified from a physical standpoint since the resonator fields respond to perturbations by attempting to minimize the roundtrip losses. Thus, decreased aperture losses at the center of the mirrors should result in decreased diffraction losses at the mirror edge.

As an example, consider the quasi-optical resonator used in the calculation of Fig. 2, where both the input and output coupling holes are 1.8 mm in diameter. The diffraction theory code result is  $\beta_1 = \beta_2 = 0.433$  and  $\beta_3 = 0.784$ , assuming that all radiation incident on the aperture is transmitted through the mirror. From Fig. 4, the measured transmission for a 1.8-mm diameter hole is  $-2.7 \text{ dB}$ . The modified coupling parameters are now  $\beta'_1 = \beta'_2 = 0.233$  and  $\beta'_3 = 0.422$ , where the prime indicates that the effect of the small aperture has been included. Using (3) and (4), the reflected power on resonance is calculated as  $R = 0.57$ , while the transmission through the far coupling hole is  $T = 6\% (-12.2 \text{ dB})$ . The remaining power (0.37) is coupled to the mode of interest in the resonator, where the loaded  $Q$  factor is now  $Q_L/Q_0 = 0.53$ .

The largest source of error in the diffraction theory model concerns the aperture transmission calculation. In this simplified model, it is assumed that  $\beta'_1$  and  $\beta'_2$  vary according to the aperture transmission measurements performed in rectangular waveguide. In the experiment, the aperture is terminated by the resonator, thus, the true transmission values may be somewhat different.

#### B. Equivalent-Circuit Model of Resonator Coupling

The equivalent-circuit model used in this paper is related to the theory of Mongia and Arora [5], where the modified form of Bethe's small aperture coupling theory was used to construct the equivalent circuit. The incident  $TE_{10}$  mode in the rectangular waveguide excites a tangential magnetic dipole at the circular aperture. This magnetic dipole then excites the  $TEM_{000}$  mode in the quasi-optical resonator. In this paper, the coupling holes are large and Bethe's small aperture theory is not valid.

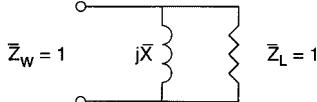


Fig. 5. Equivalent circuit for an aperture in the transverse wall of a rectangular waveguide.

Instead, we use measured aperture transmission values in rectangular waveguide to calculate an effective complex impedance of the coupling hole. The input and output coupling coefficients are then computed, which yield the reflection and transmission through the open resonator.

Consider first the calculated external  $Q$  of an open resonator with a coupling hole [5]

$$Q_e = \frac{[1 + (ab/2k_{10}\alpha_m)^2] 2k_{10}w^2(L/2)}{abA_1^2w_0^2}. \quad (5)$$

Here,  $a \times b$  are the rectangular waveguide dimensions,  $k_{10} = \sqrt{(2\pi/\lambda)^2 - (\pi/a)^2}$  is the propagation constant in the waveguide,  $\alpha_m$  is the magnetic polarizability of the aperture,  $w_0$  is the radiation waist at the center of the resonator,  $w(L/2)$  is the spot size at the mirror, and  $A_1^2 = 4/L\pi w_0^2$  is a normalization constant. The equations describing the fundamental Gaussian mode in the open resonator with mirror curvature  $R$  and mirror separation  $L$  are

$$w_0^2 = \frac{\lambda}{2\pi} \sqrt{L(2R - L)} \quad (6)$$

$$w^2(z) = w_0^2(1 + z^2/z_0^2) \quad (7)$$

$$z_0 = \pi w_0^2/\lambda. \quad (8)$$

For small apertures (hole radius  $r_0 \ll b$ ) in a zero-thickness wall, the magnetic polarizability is written

$$\alpha_m = 4r_0^3/3. \quad (9)$$

An aperture in the transverse wall of a rectangular waveguide can be modeled with the equivalent circuit shown in Fig. 5, where  $Z_w$  and  $Z_L$  are the normalized impedances of the waveguide and load, respectively. The normalized reactance of the aperture is given as

$$\bar{X} = 8k_{10}r_0^3/3ab. \quad (10)$$

The relations given above may be used to rewrite the external  $Q$  of the open resonator as

$$Q_e = \frac{[1 + \bar{X}^{-2}] 2k_{10}w^2(L/2)}{abA_1^2w_0^2}. \quad (11)$$

The power transmitted to the load in the circuit in Fig. 5 is calculated as

$$T = \frac{4\bar{X}^2}{1 + 4\bar{X}^2}. \quad (12)$$

In this paper, where we couple to low- $Q$  quasi-optical resonators, the coupling holes are large ( $2r_0 \geq b$ ) and the wall thickness is finite so that the above relations are not valid. The

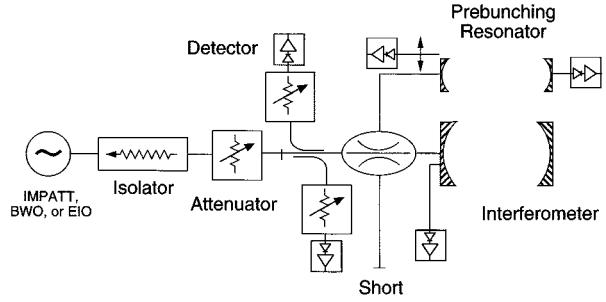


Fig. 6. Schematic diagram of the cold-test setup.

technique used in this section is to calculate an effective aperture reactance  $\bar{X}'$  based on the transmission measurements in Fig. 4. As an example, the transmission through a 1.8-mm-diameter aperture in  $W$ -band waveguide at 85.5 GHz is measured as  $-2.7$  dB. The normalized effective reactance of this aperture is calculated as  $\bar{X}' = 0.54$  from (12). For the resonator under study, the calculated waist is  $w_0 = 0.92$  cm with a mirror spot size  $w(L/2) = 1.01$  cm. The effective aperture reactance may then be substituted into (11), resulting in a calculated external  $Q$  factor  $Q_e = 20000$ . The unloaded  $Q$  of the resonator is approximately 6000 when diffraction and ohmic losses are considered, yielding an input coupling coefficient  $\beta_1 = Q_0/Q_e = 0.3$ . If the resonator is symmetric,  $\beta_1 = \beta_2$  and the increased diffraction can be scaled as  $\beta_3 \simeq 0.54$ . From (3) and (4), the resonator transmission and reflection are calculated as  $T = -12.2$  dB and  $R = 0.52$ . These values are in relatively good agreement with those calculated in the previous section using the diffraction theory model. In Section III, experimental measurements of resonator transmission and reflection will be compared to calculations from the two coupling models.

### III. COLD-TEST MEASUREMENTS

A schematic diagram of the cold-test apparatus is shown in Fig. 6, where most elements are standard WR10 waveguide components. Three sources are used in the cold tests: a 94-GHz IMPATT, a 75–110-GHz backward-wave oscillator (BWO), and an 85-GHz extended interaction oscillator (EIO). The frequency sweep of the RF source is calibrated using the high- $Q$  interferometer ( $Q = 70000$ ). The prebunching resonator has centered coupling holes for reflection and transmission measurements. A short-circuited section of waveguide is used as a 100% reference reflection, with several attenuators to reduce the voltage standing-wave ratio (VSWR) in the line. The diffraction pattern around the edge of the input resonator mirror is measured using a WR10 waveguide pickup.

Cold tests of quasi-optical resonators (low- $Q$  resonators in particular) are more difficult to interpret than closed cavities because there are more loss mechanisms. The  $Q$  of the unloaded resonator is dominated by diffraction of the  $TEM_{00}$  mode around the mirrors. The power reflected from the resonator off resonance is not necessarily 100%, which contrasts with the closed-cavity result. There is also the possibility that radiation at the resonant frequency, which is transmitted past the input aperture, is nonresonantly scattered out of the resonator. It has been shown in the previous section that the diffraction loss

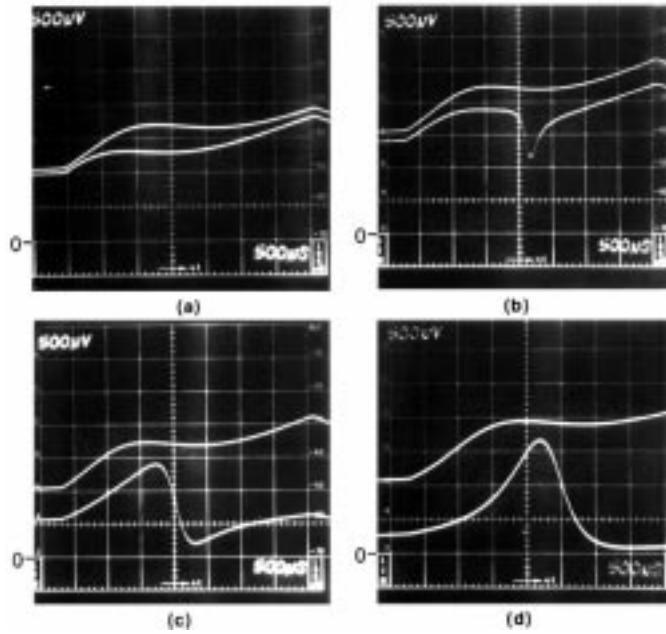


Fig. 7. (a)–(d) Reflection measurements from a low- $Q$  open resonator for coupling hole diameters of 0.91, 1.32, 1.60, and 1.85 mm, respectively. The horizontal scale is 500  $\mu$ sec/div, the vertical scale is 500  $\mu$ V/div.

is dependent upon the aperture losses. This implies that it is difficult to vary only one resonator parameter at a time during cold tests.

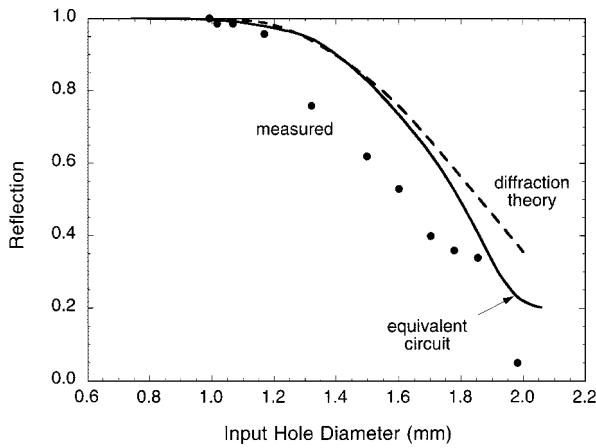
Four representative oscilloscope traces of reflection signals from a low- $Q$  open resonator are shown in Fig. 7, where the mirror diameter is 3.15 cm, the separation is 3.7 cm, the mirror radius of curvature is 20.3 cm, and the frequency is 95 GHz. The top trace in each oscilloscope is the 100% reference reflection from the short circuit, while the lower signal is the reflection from the input resonator. For a hole diameter of 0.91 mm, the input coupling is small, and there is no observed change in the reflected signal due to the resonator. The characteristic resonant dip in the reflected signal appears for a hole diameter of 1.32 mm. Note that the reflection off resonance remains at 100% for this aperture. As the input hole diameter is increased further, a large asymmetry appears in the reflected signal. This is due to interference effects caused by the phase shift in the resonator. The reflection off resonance is much less than the 100% value observed for smaller hole sizes. For a hole diameter of 1.85 mm, the resonator is overcoupled and approximately 100% of the power is reflected on resonance. Little power is reflected off resonance, which indicates that radiation is scattered nonresonantly past the mirrors.

One of the main conclusions from the data in Fig. 7 is that a large fraction of the incident power can be coupled to the  $TEM_{00}$  mode. From oscilloscope [see Fig. 7(b)], approximately 62% of the power is reflected on resonance with 100% reflection off resonance. It is reasonable to expect that most of the remaining 38% of the incident power is coupled to the mode of interest in the resonator. As the hole diameter is increased further, it is likely that the input coupling will continue to increase. A direct measurement of the strength of the field in the prebunching resonator is performed in the next series of cold tests.

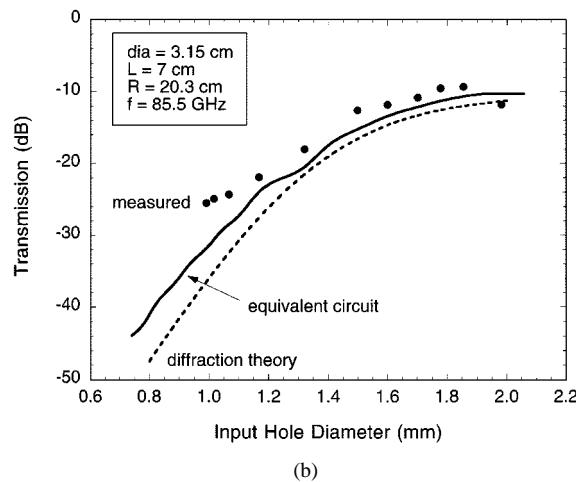
The oscilloscope traces in Fig. 7 are quite similar to the theoretical calculations presented in [13]. In that study, analytical expressions for the resonator fields were used to calculate the reflection from a waveguide coupled to a high- $Q$  quasi-optical resonator as the input aperture was varied. The calculated input admittance depends upon three contributions: the input aperture, the resonant mode, and the power radiated into free space. For small holes, the reflection coefficient is dominated by the effects of the aperture and resonant mode. The calculated reflection is similar to Fig. 7(b), where there is a dip in the reflected signal at the resonant frequency. For large coupling holes, the admittance of the aperture is small, so that the reflection depends upon the resonant mode and free-space radiation. Here, the reflection is peaked at resonance and is small off resonance [as in Fig. 7(d)]. Intermediate-sized holes result in asymmetric waveforms of the shape in Fig. 7(c), where all three effects are important. Hence, there are many qualitative similarities between the calculations of high- $Q$  resonator reflections and the present measurements on low- $Q$  resonators.

It is instructive to examine the resonator of Fig. 7(d) more closely and compare the measured and calculated reflection coefficients. For a mirror separation of 3.7 cm and no coupling holes, the diffraction losses are calculated as 0.2% from the code (a low-loss resonator). The ohmic  $Q$  is given by  $Q_\Omega = d/2\sqrt{f\pi\mu_0\sigma}$ , where  $\mu_0$  is the permeability of free space,  $d$  is the mirror separation, and the conductivity of aluminum is  $\sigma = 4.2 \times 10^7$  S/m. The resulting ohmic loss is 0.2%, thus, the total roundtrip loss of the resonator without coupling holes is 0.4%. When input and output coupling holes (1.85- and 0.66-mm diameter) are considered, the diffraction theory code result is 3.1% diffraction loss, 3.0% hole loss, and  $Q = 2447$ . The corresponding coupling parameters are  $\beta_1 = 6.6$  and  $\beta_3 = 6.7$ , where  $\beta_2$  is small and can be neglected (due to the small output hole). The calculated reflection on resonance from (3) is  $R = 0.5\%$ , which is much less than the measured value of approximately 100%. This discrepancy can be attributed to the simplified coupling model, which does not consider power radiated into free space. It was shown in [13] that radiation into free space plays an important role in the reflection from a high- $Q$  resonator with a large coupling hole. Thus, the simplified coupling model is not useful in predicting the behavior of quasi-optical resonators, which are highly overcoupled ( $\beta_1, \beta_3 \gg 1$ ). However, it will be shown that the model is useful in describing the excitation of low- $Q$  resonators with moderate coupling.

The prebunching resonator mirrors used in the gyrokylystron experiment initially have a 0.8-mm-diameter input coupling hole, a 1.78-mm output hole, 0.43-mm coupling wall thickness, 3.15-cm diameter, 20.3-cm radius of curvature, and 7-cm mirror separation. Lossy Macor rings are used to absorb 85-GHz radiation which is diffracted around the mirrors. The reflection and transmission from the resonator are measured, the input coupling hole diameter is increased, and the procedure is repeated. Results of the measurements are shown in Fig. 8, where the solid points correspond to measured values, the solid curve is the equivalent circuit model, and the dashed curve is the diffraction theory model. For input holes below 1 mm in diameter, essentially all of the power is reflected and only a small transmitted resonance is observed. As the input aperture



(a)



(b)

Fig. 8. (a) Reflected and (b) transmitted signals to the open resonator versus input hole diameter.

is increased, the transmitted signal increases monotonically by 17 dB. The transmitted resonance is symmetric in shape as the frequency is swept, and there is very little background radiation detected off resonance. This indicates that the signal from the far coupling hole is due to the  $TEM_{00}$  mode with little contribution from nonresonant radiation. The maximum transmitted signal occurs for a coupling hole diameter of 1.85 mm, resulting in a reflection  $R = 0.36$  and a transmission  $T = -9.3$  dB. Increasing the size of the coupling hole past this point decreases the reflection, but also decreases the power coupled to the  $TEM_{00}$  mode. This result is quite different from the closed-cavity case, and indicates that power is transmitted past the input aperture, but is not coupled to the mode of interest in the resonator.

At the optimum coupling point, the equivalent circuit model predicts  $R = 0.42$  and  $T = -10.7$  dB, which are close to the measured values given above. The equivalent-circuit model is in better agreement with the measured data, which may be due to approximations in the diffraction theory model. Scalar diffraction theory calculates the coupling coefficients  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  very accurately, but the attenuation values through the coupling holes are approximate. From Fig. 8, the measured reflection is consistently less than the calculated values, whereas the measured transmission is greater than the theory. This indicates that

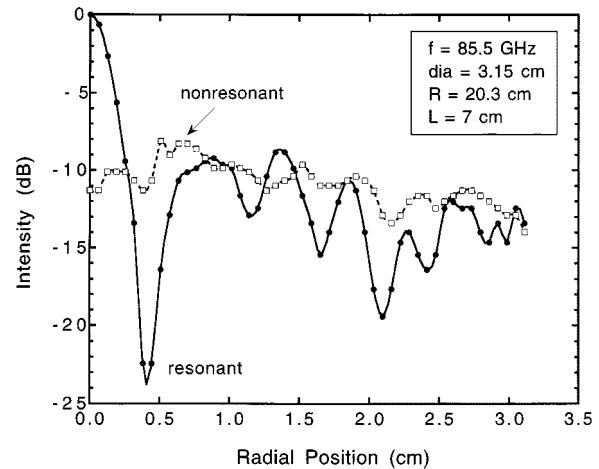


Fig. 9. Measured radial scans of the diffraction from the open resonator.

there is more power coupled to the desired  $TEM_{00}$  mode than the models predict. This is somewhat surprising since one would expect that including higher order modes and scattered radiation in the models would adversely affect the coupling. For an input hole of 1.85-mm diameter, the transmission has a well-defined peak and decreases for larger holes. This effect is not observed in the models, and may be related to the cutoff property of the aperture in the experiment. The optimum hole diameter of 1.85 mm is close to the cutoff diameter of 2.06 mm for 85.5-GHz radiation in the  $TE_{1,1}$  mode in the circular aperture.

A section of WR10 waveguide (without the flange) is used to measure the diffraction pattern of the  $TEM_{00}$  mode past the mirror edge. The measurements are performed for the resonator described above with optimized input coupling. Also shown in Fig. 9 is the radiation pattern past the mirror edge when the frequency of the RF source is shifted off resonance by a small amount. The position 0.0 cm denotes the edge of the mirror and the waveguide pick-up is translated in the vertical direction in the plane perpendicular to the axis of the resonator. The  $TEM_{00}$  mode has a large peak at the mirror edge and a series of sidelobes. The first minima is approximately 23 dB down from the intensity at the edge of the mirror, and is followed by sidelobes of approximately -10 dB. However, the nonresonant scan shows none of the structure associated with the fundamental mode. The radial scans confirm that most of the incident power is coupled to the mode of interest and is not scattered from the resonator.

Calculations are performed to compare electric-field profiles generated by the scalar diffraction theory code to the measured intensity pattern in Fig. 9. However, the computer code cannot simultaneously compute the fields in both the aperture and region outside the mirror edge. Hence, the present comparison is qualitative rather than quantitative, where the calculated patterns are computed without the centered coupling hole. Two representative radiation patterns are plotted in Fig. 10 for mirror diameters of 3.16 and 2.852 cm, which correspond to roundtrip diffraction losses of 4.0% and 11.9%, respectively. As in the previous figure, the position 0.0 cm corresponds to the mirror edge. However, the fields are calculated along a surface with the same curvature as the mirror ( $R_c = 20.3$  cm). The radial scale for

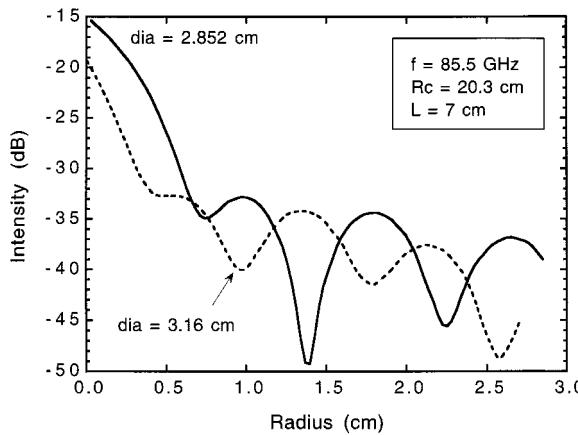


Fig. 10. Calculated diffraction patterns of the resonator for two values of mirror diameter.

the measured and calculated intensity profiles is approximately equal when the mirror curvature is much larger than the distance from the mirror edge. The intensity in Fig. 10 is referenced to the peak intensity at the center of the mirror. Typical calculated sidelobe levels are 13–18 dB down from the peak at the edge of the mirror, and vary with the size of the mirrors. There are a number of similarities between the measured and calculated patterns, although it must be emphasized that the calculations do not consider the effect of the (large) coupling holes. It should also be noted that the diffraction theory is based on two-dimensional calculations, whereas the diffraction pattern from the real resonator is inherently three dimensional.

#### IV. CONCLUSIONS

A low- $Q$  open resonator has been successfully designed and tested as the input resonator in a quasi-optical gyrokylystron experiment at 85 GHz. The fundamental  $TEM_{00}$  mode is the dominant mode for large coupling holes provided that the coupling losses are somewhat less than the diffraction losses. Cold-tests measurements indicate a loaded  $Q = 2000$  with approximately 65% of the incident power coupled to the fundamental transverse mode. The roundtrip losses are 13%, where coupling hole losses account for 3.8%. This is the first study of open resonators with large diffraction losses and efficient input coupling.

Two approaches have been used to model the coupling from the rectangular waveguide through the circular aperture into the low- $Q$  open resonator. The first is based on scalar diffraction theory, where a computer code is used to calculate the diffraction and coupling losses of the resonator. It has been shown that introducing coupling hole loss increases the diffraction loss at the edge of the mirrors. This effect has been included in both models, and results in markedly different coupling behavior compared to conventional closed cavities. Aperture transmission measurements are performed in  $W$ -band waveguide at 85.5 GHz to approximate the transmission through the circular coupling holes in the mirror. The second technique uses an equivalent-circuit model of the resonator and aperture

to calculate the external  $Q$  due to the coupling holes. Since the coupling holes are large and the wall thickness is appreciable, the small aperture approximations of Bethe are not valid for this paper. Instead, the transmission measurements in the waveguide are used to define an effective reactance of the aperture, which is then substituted into the model. Calculated results from the two models are in relatively good agreement with reflection and transmission measurements on the quasi-optical resonator. It is interesting that the measured transmission is typically 1.5 dB greater than the theory, which indicates that losses not considered in the model (such as scattering and coupling to higher order modes) are not important in this regime. The general properties of these quasi-optical resonators are much different from closed cavities because the diffraction losses depend upon the coupling apertures and there are more loss mechanisms.

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**Richard P. Fischer** (S'83–M'84) received the B.S., M.S., and Ph.D. degrees in electrical engineering from the University of Maryland at College Park, in 1984, 1986, and 1993, respectively. His Ph.D. dissertation concerned the design and test of a high-power millimeter-wave quasi-optical gyrokylystron.

From 1984 to 1986, he was a Research Assistant in the Solid State Laser Laboratory, University of Maryland at College Park. From 1986 to 1988, he was an electronics engineer with Jaycor, where he was involved with the staff of the Naval Research Laboratory (NRL), Washington, DC, on gyrotron experiments and diagnostics. Since 1988, he has been with the Beam Physics Branch, NRL, where his research interests include high-power gyrotrons, the interaction of intense laser beams with electron beams and plasmas, and materials processing using high-power millimeter-wave and laser radiation.

Dr. Fischer is a member of the American Physical Society.



**Arne W. Fliflet** (M'86) received the B.Sc. degree in physics from Duke University, Dunham, NC, in 1970, and the Ph.D. degree in physics from the University of Virginia, Charlottesville, in 1975.

From 1975 to 1979, he was a Research Fellow at the California Institute of Technology, where he was involved with theoretical approaches to low-energy electron–molecule collision processes. From 1979 to 1982, he was a Project Scientist with B-K Dynamics Inc. Rockville, MD, where he developed theoretical models for gyrotron oscillators. In 1982, he joined

the Naval Research Laboratory, Washington, DC, as a Research Physicist in the High Power Electromagnetic Radiation Branch, where he has conducted research in areas of gyrotron theory and experiment. He is currently Head of the Radiation and Particle Beam Generation Section. Current research interests include conventional and QOGs and their application to microwave processing of materials, free-electron lasers, and magnetron amplifiers for powering the next linear collider.

Dr. Fliflet is a member of the IEEE Nuclear and Plasma Sciences Society and a Fellow of the American Physical Society.